Structural Optimization with Frequency Constraints—A Review

Ramana Grandhi
Wright State University, Dayton, Ohio 45435

I. Introduction

SEVERAL survey papers exist in the field of structural optimization[1,9,55,129,138,157] covering optimization algorithms, constraint approximations, sensitivity analysis, shape optimization, multilevel decomposition, etc. This paper covers structural optimization with dynamic frequency constraints because of its importance in structural design. A large number of publications have appeared on this topic, covering a wide variety of related problems such as flutter control, buckling, structure/control design, etc. The following topics are addressed in this review: how the frequency considerations influence the structural design, their sensitivity, and constraint approximations, and how different types of optimization algorithms are used in solving the frequency problem.

The optimal design of structures with frequency constraints is extremely useful in manipulating the dynamic characteristics in a variety of ways. For example, in most low-frequency vibration problems, the response of the structure to dynamic excitation is primarily a function of its fundamental frequency and mode shape. In such cases, the ability to manipulate the selected frequency can significantly improve the performance of the structure. Similarly, the aeroelastic characteristics of an aircraft wing are governed primarily by its torsional and bending properties, which can best be studied by the lower torsional and bending modes. In aircraft design, frequency requirements are often imposed on control surfaces and major structural components as flutter prevention measures. In designing most space vehicles, it is highly desirable to restrict several of the lower mode structural frequencies to prescribed ranges to avoid coupling with the control system. In fact, in most narrow-band excitation problems, controlling the frequencies in the critical range is tantamount to controlling the dynamic response.

The most common problem in frequency optimization seems to be the switching of vibration modes due to structural size modifications. One may not be optimizing or following the same bending, torsion, or axial mode as the design changes. Mode switching causes convergence difficulties to the optimizer. In some cases, such as in turbomachinery, it is not clear whether the design modifications switched the mode shapes or the structural sizes triggered multiple frequency paths beyond the bifurcation point. Another concern is that the amount of material that would be necessary to attain an arbitrarily prescribed frequency can be disproportionally high, thus yielding a design that is completely impractical. In some cases, an upper bound on the maximum frequency can be attained for a given configuration irrespective of the amount of material used. Finally, the structural weight minimization results in very closely spaced natural frequencies, and their range decreases considerably. Some structures at the optimum designs exhibit repeated eigenvalues even though the initial design did not have any. The closeness and repeated nature of eigenvalues pose a tremendous challenge for the controls engineer.

This review article is organized as follows based on the published journal articles covering relevant disciplines: Sec. II states the different forms of problem statements used in frequency optimization; Sec. III discusses the sensitivity derivatives calculation, particularly for repeated eigenvalues; Sec. IV describes the constraint approximations developed specifically for frequency constraints; Sec. V presents different types of optimization algorithms used in solving the mathematical problem; Sec. VI gives frequency-related disciplines such as helicopter vibration reduction, buckling, and closed-loop eigenvalues of the active control system; and, finally, Table 1 categorizes and lists the previous work by the type of structural applications.

II. Problem Statement

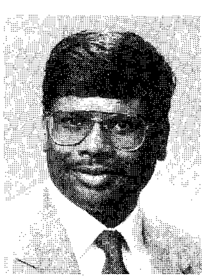
The structural optimization problem with frequency constraints is posed in one of the following two ways:

1) Minimize structural weight $f(x)$ subject to behavior constraints

$$\begin{aligned} g_j(x) &= \omega_j^2 - \tilde{\omega}_j^2 = 0 \qquad j = 1, 2, \dots, k \\ g_j(x) &= \omega_j^2 - \tilde{\omega}_j^2 \geq 0 \qquad j = k+1, k+2, \dots, m \end{aligned} \tag{1}$$

Table 1 Frequency optimization literature by area of application

Type of problem	References
Arches	15,119,132,133
Beams/frames	7,16,17,28,40,50,60,78,80,86,109,131,147,152,155,181
Composites	14,20,85,87,93,99,140,160
Disks	36,37
Membrane	21,22,51,59,143,166
Plates	6,43,45,53,54,82,98,125,144
Shells	18,19,81,122,123,137,141,142
Truss	13,21,22,42,49,73,76,83,86,106,149,188
Blades	29,31,32,48,127



Ramana Grandhi received his Ph.D. from Virginia Polytechnnic Institute and State University in 1984, then joined Wright State University as an Assistant Professor. He received the Ralph R. Teetor Award from the Society of Automotive Engineers in 1987, DOW Outstanding Faculty Award from the American Society of Engineering Education in 1989, Up & Comers Award from Price Waterhouse in 1990, and currently he is Brage Golding Distinguished Research Professor and an Associate Fellow of AIAA.

Presented as Paper 92-4813 at the AIAA/USAF/NASA/OAI Symposium on Multidisciplinary Analysis and Optimization, Cleveland, OH, Sept. 21–23, 1992; received Oct. 3, 1992; revision received July 13, 1993; accepted for publication July 20, 1993. This paper is declared a work of the U.S. Government and is not subject to copyright protection in the United States.

2) Maximize the fundamental frequency or difference between two consecutive frequencies subject to a specified weight constraint

$$g(x) = f(x) - \tilde{f} = 0 \tag{2}$$

and side constraints on the design variables

$$x_i^l \leq x_i \leq x_i^u \tag{3}$$

where x_i is the design variable, x_i^l is the lower limit, x_i^u is the upper limit on the design variable, ω_j is the jth natural frequency, $\tilde{\omega}_j$ is the specified value of the jth frequency, $f(x)$ is the structural weight, $\tilde{f}$ is the specified weight, n is the number of design variables, and m is the number of design constraints.

The design variables depend on the type of optimization problem. In the design of structural components, such as stiffened panels and cylinders, the design variables represent the spacing of the stiffeners, the size and shape of the stiffeners, and the thickness of the skin. If the skin and/or stiffeners are made of layered composites, the orientation of the fibers and their proportion can become additional variables. In the optimization of a structural system (frames, trusses, wings, fuselages, etc.) of fixed configuration, the sizes of the elements are design variables. The thickness of plates, cross-sectional areas of bars, areas, moments of inertia, and torsional constants of beams represent sizes of the elements. If the optimization includes configuration, the variables include spatial variables. Also, in dynamic problems, the location of nonstructural masses and their magnitudes can be considered as variables.

If only frequency constraints are considered in the optimization problem, it is advisable to include nonstructural masses in the structural model representing the fuel, payload, attachments, etc. Also, in the earlier problem statement, when the objective is posed as the maximization of the higher order frequency or the difference between two consecutive frequencies, additional frequency constraints may have to be included to prevent driving all of the lower frequencies to a zero value. Finally, the designs become more practical with additional constraints such as stresses and displacements under multiple load conditions.

III. Sensitivity Analysis

Sensitivity derivatives are used for studying the effect of parametric modifications, calculating the search directions for finding an optimum design, constructing function approximations, and conducting what-if tradeoff design studies. Recent surveys presented in Refs. 1 and 57 thoroughly cover the developments and applications of the sensitivity analysis. In this section, sensitivity analysis tools used in frequency optimization are discussed.

The eigenvalue problem is given as follows:

$$\boldsymbol{K}\boldsymbol{u} = \lambda \boldsymbol{M}\boldsymbol{u} \tag{4}$$

where $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{M}$ is the mass matrix, λ is the eigenvalue, and $\boldsymbol{u}$ is the eigenvector. The derivatives of the distinct eigenvalues using the orthogonality conditions are given as

$$\lambda' = \boldsymbol{u}^t(\boldsymbol{K}' - \lambda \boldsymbol{M}')\boldsymbol{u} \tag{5}$$

where a prime denotes a derivative with respect to the design variable.

Fox and Kapoor[46] presented methods for calculating the eigenvalue and eigenvector derivatives of symmetric matrices. Two methods were presented for eigenvector gradients. In the first method, the algebraic eigenvalue problem was differentiated with respect to the design variables, and after the algebraic manipulations were carried out, the derivatives were calculated. The banded nature of the equations was disturbed in this approach. In the second method, the derivative was expanded as a series of eigenvectors. Later, Nelson[108] developed an alternate procedure for the eigenvector derivatives while retaining the banded nature of matrices. Pritchard et al.[135] developed an expression for the derivative of the nodal location of the mode shape with respect to the design variable for one-dimensional structures. Sutter et al.[162] compared four methods for calculating the derivatives of vibration modes with respect to the design parameters. They concluded that Nelson's method was superior because it was exact and required less computer time.

The sensitivity calculation of repeated eigenvalues has been investigated by several researchers. Repeated eigenvalues are not differentiable, and only directional derivatives can be found. References 10, 33, 64, and 105 solved structural optimization problems with repeated eigenvalues by employing directional derivatives. For the real symmetric case, a generalization of Nelson's method, which preserves the bandedness of the matrix, was presented in Refs. 34, 101, and 111. The complications in sensitivity computation are related to the fact that the eigenvectors of the repeated eigenvalues are not unique. The eigenvalue derivatives for repeated roots can be found by solving a subeigenvalue problem

$$[\Phi^t(\boldsymbol{K}' - \lambda \boldsymbol{M}')\Phi - \lambda' \boldsymbol{I}]\boldsymbol{a} = 0 \tag{6}$$

where Φ consists of eigenvectors corresponding to repeated roots, $\boldsymbol{a}$ is a coefficient vector, and $\boldsymbol{I}$ is an identity matrix. The eigenvalues of Eq. (6) represent the λ' vector.

Reduced-order models were used in computing sensitivities for both the repeated and nonrepeated frequencies in Ref. 30. Hou and Chuang[65] developed eigenvalue and eigenvector sensitivity equations of continuous beams subjected to the variations of support locations. Both the domain and the boundary methods were employed in their derivations.

IV. Constraint Approximations

A recent paper by Barthelemy and Haftka[9] reviewed function approximations used in structural optimization by classifying them as local, medium-range, and global types. Most of the generally available approximation techniques are applicable to frequency functions, but several researchers developed high-quality approximations just for frequency problems[23,102,160,175,187] to get a stable convergence with less restrictive move limits.

Because of the inherent nonlinear characteristics of natural frequency constraints, Miura and Schmit[102] employed a second-order Taylor series approximation for each eigenvalue for improving the stability and overall efficiency of the synthesis process. Their studies revealed that the eigenvalues are highly nonlinear in both direct and reciprocal design variable space, requiring strict move limits. Although the second-order approximation provided stable convergence without strict move limits, they reported that the total computational time was "comparable with that required using first-order approximation with move limits." Starnes and Haftka[160] and Fleury and Braibant[44] have shown that a hybrid constraint using mixed variables (i.e., a combination of direct and reciprocal variables) yields a more conservative approximation. Woo[187] generalized the concept in his generalized hybrid constraint (GHC) approximation, where a variable exponent controls the conservativeness of the convex approximation, and demonstrated the concepts on a space frame structures design. Second-order approximations using a half-quadratic scheme, a generalized power approach, a generalized method of moving asymptotes, and a full second-order Taylor approximation were presented in Ref. 103. Based on the computational cost, the authors concluded that the second-order approximations may be used for difficult-to-solve problems, and for less sensitive problems the approximate second-order information is recommended. Pritchard and Adelman[136] presented an interesting approach by interpreting the sensitivity expressions as differential equations and obtained closed-form exponential approximations for eigenvalues and eigenvectors. The quality of approximations was better than the linear models.

Even though the nonlinearity of frequencies is readily observed through the appearance of cross-sectional variables in both the numerator and denominator of Rayleigh's quotient, Venkayya and Tischler[178] have pointed out that, in practical structures, the denominator (kinetic energy) is typically dominated by the nonstructural mass. In this case, the eigenvalues are more nearly linear in the cross-sectional property (direct design variable space). Vanderplaats and Salajegheh[175] demonstrated improved quality using a linear approximation of the eigenvalues with respect to the member section properties of frame elements when the optimization design variables were cross-sectional dimensions. No attempt was made to create a convex or separable form of the optimization problem.

The optimality criterion approach presented by Venkayya and Tischler[178] and Grandhi and Venkayya[49,51] suggest that the modal strain and kinetic energies may be better quantities to approximate than the eigenvalue. Canfield[23] developed the Rayleigh quotient approximation (RQA) by constructing first-order approximations to the modal strain and kinetic energies independently:

$$\lambda = \frac{u^t K u}{u^t M u} = \frac{U_A}{T_A} \tag{7}$$

where U_A and T_A are the first-order approximations for modal strain and kinetic energies, respectively. He obtained fast and stable convergence with generous move limits. In fact, the concept is similar to an alternative approximation proposed by Fox and Kapoor,[46] except that here the eigenvector's first-order estimate was not used.

V. Algorithms Used in Frequency Optimization

Design optimization, unlike analysis, is a multivalued problem. In general most optimization problems do not have a unique solution. The usual procedure is to establish a set of necessary and sufficient conditions for an optimum and then construct an algorithm that systematically leads to a solution that satisfies such conditions. The uncertainty about the nature of the absolute optimum fostered the development and improvement of numerous optimization algorithms. Several methods are being developed and used in solving the frequency optimization problem, and these are broadly classified as 1) mathematical programming (MP) techniques, 2) optimality criteria (OC) methods, 3) optimal control techniques, and 4) other methods.

This classification can be quite nebulous because there can be a great deal of overlapping due to the fact that the concepts developed in one method are being adopted in the other methods for improving the efficiency and convergence stability. For example, in mathematical programming, a standard optimization algorithm is established, then the constraint approximations are constructed for the behavior functions, and the design variables are updated iteratively by using search directions. In the OC approach, first the optimality criterion is derived, and it is satisfied recursively by constructing an algorithm making use of function approximations based on the nature of active constraints. The work of Fleury and Braibant[44] utilizes concepts of both methods in developing dual methods.

The mathematical programming methods include linear, nonlinear, geometric, and integer programming methods. In this review, the OC methods include the variational approach used in deriving the optimality criterion in an integral form for the continuous problems. Application of the optimal control theory is very limited in structural optimization, and not much activity appeared in this area during the 1980s.

A. Mathematical Programming Techniques

Frequency optimization problems were solved by various mathematical programming methods. The penalty function methods were used the most[13,19,27,50,72,139,143] among these methods. Other methods such as feasible directions,[42,47,85,123] gradient projection method,[190] multiplier algorithms,[161] sequential linear programming,[32,126] sequential quadratic programming,[31] etc., were also used. Felix and Vanderplaats[42] presented optimum configuration designs of three-dimensional trusses using the multilevel optimization technique with stress, displacement, Euler buckling, and natural frequency limits. The optimum configurations were strongly dependent on the type of constraints imposed on the design. Performance of sensitivity analysis methods[172] and optimization algorithms [recursive quadratic programming, feasible directions, gradient projection, sequential unconstrained minimization technique (SUMT), and multiplier methods] using multiple constraints including frequency were compared in Refs. 12 and 21 with the goal of identifying reliable and efficient methods.

Recently, the sensitivity analysis and frequency optimization are carried out by using the large-scale, general purpose, finite element-based multidisciplinary computer programs. Neill et al.[107] discussed the development and use of the automated structural optimization system (ASTROS) with wing structure examples. Somayajula and Bernard[158] used MSC/NASTRAN sensitivities in optimizing the plate and pipe problems. Kodiyalam et al.[80] developed the shape and size design optimization capabilities in MSC/NASTRAN and demonstrated on various types of structural applications. The most recently developed GENESIS[170] code was designed and written from the start to perform structural design in the multidisciplinary environment. The feasible directions algorithm was used with constraint approximations and move limits in both the computer programs. Among the mathematical programming methods, the feasible directions algorithm is still widely used with state-of-the-art constraint approximations because of its efficiency and robustness in solving the multidisciplinary optimization problems.

B. Optimality Criteria Methods

In this subsection, the previous work done using the optimality criteria is divided into two categories: the continuous approach (distributed systems) and the discrete approach. The resulting integrodifferential equations and their solutions are continuous functions of spatial variables in continuous approach, whereas the equations are algebraic in discrete approach.

Continuous Approach

Derivations of the optimality equations using the variational methods deal with the extremum of a function of functions. The resulting solution is not an extremum point, but one or more functions, which are represented by differential equations. The solutions of these differential equations represent the optimal path or all optimal points in the domain of definition. Most often they are nonlinear, strongly coupled integrodifferential equations, and finding the solution in closed form is difficult except in the case of very simple problems. Multidisciplinary design as an optimization problem becomes even more intractable in the context of variational calculus.[180]

Niordson[110] was the first one to use this approach in demonstrating that the fundamental frequency of transverse vibration can be maximized by tapering the beam. Following this work, several researchers used the distributed systems approach for solving beam and plate problems with various boundary conditions.[5,17,25,26,39–41,70,71,153,167,168,183] Olhoff[112–118] solved these equations numerically by a successive iteration procedure based on a finite difference discretization. The maximization of higher order frequencies, sensitivity of the eigenvalue relative to geometric imperfections, and consideration of interior support positions as design variables were presented by Olhoff.[116] The maximization of the difference between the adjacent frequencies was also addressed by this approach.[120]

Using the calculus of variations approach, researchers formulated a bimodal optimization problem to maximize the fundamental frequency of shallow arches.[119,132,133] A limit to the maximum attainable fundamental frequency is associated with any given minimum constraint value for the cross-sectional height without a further increase in the arc length.[119] Necessary and sufficient conditions for the optimum design with repeated eigenvalues were presented by Masur.[91,92]

Discrete Approach

As evidenced from the literature, the OC approach for the structures modeled with finite elements has been used widely.[22,67–69,83,95,146,148–150,159,174,182,184]

The OC algorithm presented by Venkayya et al.[177] consisted of three fundamental steps: structural analysis, scaling, and resizing. Several other researchers did not employ the scaling step,[74,79,165,189] but they directly used the resizing algorithm. A recurrence relation derived from the optimality criterion is used to modify the design variables. The optimality criterion is obtained by differentiating the Lagrangian with respect to the design variables. The essence of the optimality criterion is that at the optimum the weighted sum of the Lagrangian energy density must be the same in all of the elements. The optimality condition consists of the gradients (constraints and objective function) and Lagrange multipliers. In the case of a single constraint, the evaluation of the Lagrange multiplier was avoided by assuming it to be equal to unity,[178] and the designs were scaled to the feasible region. However, when more than one constraint is active, the Lagrange multipliers were computed for achieving better convergence. Khot[76] used a set of simultaneous linear equations to determine the Lagrange multipliers. Grandhi and Venkayya[49,51] approximated these values by using simple expressions derived from single constraint conditions. The step size control using an adaptive procedure was presented in Refs. 96, 97, and 128. The implicit assumption in all of these papers was that the mode shapes change only insignificantly during the design cycles.

Further advances made by Venkayya and his co-workers generalized the OC methods to include multiple behavior constraints through a compound scaling technique. Grandhi et al.[53] applied the generalized compound scaling method in designing the plate structures with multiple behavior constraints where the behavior functions were nonlinear functions of design variables.

Besides the structural size variables, the finite element nodal coordinates were considered as additional design variables in Refs. 86 and 149. An algorithm suitable for repeated eigenvalues is presented in Ref. 165.

C. Optimal Control Techniques

The application of techniques based on optimal control theory has proven to be powerful when the nature of the structural member is such that its behavior can be described by an ordinary differential equation in one independent spatial variable (one-dimensional structure).[94,130,163,185,186] Relatively few applications have been made to the optimal design of structural members whose behavior is described by a partial differential equation in two independent spatial variables (two-dimensional structures).[45,61–63]

Pontryagin's principle of optimal control theory is used to derive the necessary conditions for minimum mass design satisfying the frequency constraint.[164,176] In this, an integral function is minimized by satisfying the differential equations of vibrations, boundary values, and constraints in finding the thickness distribution as a function of a spatial variable. Miele[100] used the sequential ordinary gradient-restoration algorithm (SOGRA) and the modified quasilinearization algorithm (MQA) for solving the one-dimensional problems. A generalized steepest descent method for boundary-value state equations was developed based on gradient-projection and steepest programming methods.[8,62,63] One-and two-dimensional problems were solved with multiple constraints. As pointed out in Ref. 62, the continuous optimal design approach is not directly applicable to large-scale structures with many members, but it can be used before fine discretization for detailed design.

D. Other Methods

Shin et al.[154] presented a simultaneous analysis and design approach for eigenvalue maximization for both unimodal and bimodal solutions. The nonlinear algebraic equations were solved by Powell's method. Hajela and Shih[58] used multiobjective optimization with mixed integer and discrete design variables. The global criterion approach with a branch-and-bound strategy was used. Tseng and Lu[173] solved the multiobjective problem using goal programming, compromise programming, and surrogate worth tradeoff methods where the maximization of the frequency was one of the objectives. Frequency requirements were satisfied by using inverse iteration techniques where perturbation methods were employed in finding the modified structures.[156]

VI. Frequency-Related Problems

In this section, frequency optimization issues in structures-related disciplines are discussed. For example, in the first two subsections, frequency is one of the many multiple constraints. Optimization of the buckling problem is similar to the eigenvalue problem.

A. Structure/Control Optimization

Recently, the simultaneous design of space structures and their control systems has received tremendous attention due to the reductions realized in structural weights, control efforts, and improvements in closed-loop performance. The integrated design approach optimally modifies both the structural properties and the control system characteristics to meet the stringent space structure requirements. The integrated optimization problem included design constraints on transient response, actuator forces, nonstructural mass locations and magnitudes, closed-loop eigenvalues, and damping parameters. Because of a large number of publications appearing on this subject, only some of the most recent papers that use eigenvalue constraints are listed in this review to show an active application of frequency-related optimization.[56,87–89,121,179]

Rew et al.[145] presented a pole placement technique for obtaining a robust eigenstructure using the state energy, control energy, and stability robustness measure as the objective functions. A least-squares approach was used in determining the well-conditioned eigenvectors. Similar studies were conducted by Juang et al.[66] Grandhi[52] studied the synergistic effects of passive damping on the optimum design using closed-loop eigenvalue constraints. Khot and Veley[77] presented results for minimum weight design of structures with constraints on the closed-loop frequency distribution, stability robustness of the closed-loop system, and damping parameters. Thomas et al.[169] presented high-quality approximations for complex eigenvalues, steady-state harmonic displacements, and control forces by using noncollocated control configurations. Multiobjective optimization of the minimum mass, control effort, and number of actuators was obtained using a utopian formulation.[151]

B. Helicopter Vibration Reduction

The principle source of helicopter vibrations is the main rotor. Attempts to reduce vibration levels in the rotor involves multiple disciplines (aerodynamics, dynamics, structures, and acoustics), and these are very strongly interacting disciplines. A major research thrust is under way at NASA Langley Research Center[2] for the integrated design of rotorcraft. Blade design by including frequency constraints is presented in Refs. 29, 35, 48, and 127. Analytical sensitivity derivatives were developed for blade response, hub loads, stability, and frequencies with respect to nonstructural mass and blade geometry design variables.[84] References 11 and 104 presented the helicopter airframe structures optimization problem. Adelman et al.[4] compared the optimization and experimental results in placing the tuning masses for reducing the hub shear.

C. Buckling Problems

If the geometric stiffness consists of only linear terms with respect to internal loads, then an eigenvalue problem may be solved to determine the buckling load. The linearized buckling load of a structure is given by the product of the fundamental eigenvalue and the appropriate load vector. The design goal is to find the best distribution of material that avoids global buckling. Similar to the structural eigenvalue problem, the buckling design problem may be defined as finding the minimum weight structure that satisfies a prescribed buckling load, or alternatively it may be to maximize the buckling load for a given volume or weight. Several researchers applied the structural eigenvalue optimization algorithms, constraint approximations, and sensitivity analysis tools for the buckling problems.[19,24,75,82,90,124,134]

VII. Summary Discussions

This paper presented a review of the structural optimization with frequency constraints. Table 1 presents a summary of the frequency optimization literature by the area of application. The following remarks are made:

1) Much improved and economical designs may be obtained by a simultaneous change in configuration (such as beam lengths, boundary, or support conditions) coupled with structural size modifications. Limited work was published using shape/topology considerations.

2) Not much work has been reported on the experimental validation of optimized designs with frequency requirements.[3] This is one of the areas to pursue because the optimized designs are more sensitive to the parametric uncertainties of the physical system.

3) Future research should also develop formal methods for monitoring the mode-switching phenomenon in optimization. Recent publications[38,171] utilized the modal assurance criterion, the higher order eigenpair perturbations, and the cross-orthogonality checks for tracking the modes.

4) More realistic designs can be obtained by including stress and displacement constraints under multiple load conditions in the frequency constraint problem.

5) Application of the frequency optimization in the smart structures field for vibration control needs to be addressed. Also, these problems may be investigated using neural networks and genetic algorithms.

Acknowledgments

This research work was supported by the Wright Patterson Air Force Base Contract F33615-87-C-1550 and NASA Grant NAG 3-1489.

References

[1]Adelman, H. M., and Haftka, R. T., "Sensitivity Analysis for Discrete Structural Systems," *AIAA Journal*, Vol. 24, No. 5, 1986, pp. 823–832.

[2]Adelman, H. M., and Mantay, W. R., "Integrated Multidisciplinary Design Optimization of Rotorcraft," *Journal of Aircraft*, Vol. 28, No. 1, 1991, pp. 22–28.

[3]Adelman, H. M., "Experimental Validation of the Utility of Structural Optimization," *Journal of Structural Optimization*, Vol. 5, Jan. 1992, pp. 2–11.

[4]Adelman, H. M., Walsh, J., and Pritchard, J. I., "Recent Advances in Integrated Multidisciplinary Optimization of Rotorcraft," AIAA Paper 92-4777, 1992.

[5]Akesson, B., and Olhoff, N., "Minimum Stiffness of Optimally Located Supports for Maximum Value of Beam Eigenfrequencies," *Journal of Sound and Vibration*, Vol. 120, No. 3, 1988, pp. 457–463.

[6]Armand, J. L., "Minimum Mass Design of a Plate Like Structure for Specified Fundamental Frequency," *AIAA Journal*, Vol. 9, No. 9, 1971, pp. 1739–1745.

[7]Arora, J. S., Haug, E. J., and Rim, K., "Optimal Design of Plane Frame," *Journal of the Structural Division, ASCE*, Vol. 101, ST10, 1975, pp. 2063–2078.

[8]Arora, J. S., and Haug, E. J., "Efficient Optimal Design of Structures by Generalized Steepest Descent Programming," *International Journal for Numerical Methods in Engineering*, Vol. 10, No. 4, 1976, pp. 747–766.

[9]Barthelemy, J.-F., and Haftka, R. T., "Recent Advances in Approximation Concepts for Optimum Structural Design," *NATO/DFG ASI on Optimization of Large Structural Systems* (Berchtesgaden, Germany), edited by G. I. N. Rozvany, Vol. 1, NATO ASI Series, Kluwer, The Netherlands, 1993, pp. 235–256.

[10]Bartholomew, P., and Pitcher, N., "Optimization of Structures with Repeated Normal-Mode Frequencies," *Engineering Optimization*, Vol. 7, 1984, pp. 195–208.

[11]Bartholomew, P., "Vibration Reduction for Helicopter Airframes: An Application of the General-Purpose Structural Optimization Program STARS," AIAA Paper 92-4782, 1992.

[12]Belegundu, A. D., and Arora, J. S., "A Study of Mathematical Programming Methods for Structural Optimization, Part II: Numerical Results," *International Journal for Numerical Methods in Engineering*, Vol. 21, No. 8, 1985, pp. 1601–1623.

[13]Bellagamba, L., and Yang, T. Y., "Minimum-Mass Truss Structures with Constraints on Fundamental Natural Frequency," *AIAA Journal*, Vol. 19, No. 11, 1981, pp. 1452–1458.

[14]Bert, C. W., "Optimal Design of a Composite-Material Plate to Maximize its Fundamental Frequency," *Journal of Sound and Vibration*, Vol. 50, No. 2, 1977, pp. 229–237.

[15]Blachut, J., and Gajewski, A., "On Unimodal and Bimodal Optimal Design of Funicular Arches," *International Journal of Solids and Structures*, Vol. 17, No. 7, 1981, pp. 653–657.

[16]Brach, R. M., "On the Extremal Fundamental Frequencies of Vibrating Beams," *International Journal of Solids and Structures*, Vol. 4, No. 7, 1968, pp. 667–674.

[17]Brach, R. M., "On Optimal Design of Vibrating Structures," *Journal of Optimization Theory and Applications*, Vol. 11, No. 6, 1973, pp. 662–667.

[18]Bratus, A. S., "Design of Circular Cylindrical Shells of Minimum Weight with Fixed Natural Oscillation Frequencies," *Journal of Applied Mathematics and Mechanics*, Vol. 47, No. 5, 1983, pp. 650–657.

[19]Bronwicki, A. J., Nelson, R. B., Felton, L. P., and Schmit, L. A., Jr., "Optimization of Ring Stiffened Cylindrical Shells," *AIAA Journal*, Vol. 13, No. 10, 1975, pp. 1319–1325.

[20]Bruch, J. C., Adali, A., Sloss, J. M., and Sadek, I. S., "Optimal Design and Control of a Cross-Ply Laminate for Maximum Frequency and Minimum Dynamic Response," *Computers and Structures*, Vol. 37, No. 1, 1990, pp. 87–94.

[21]Canfield, R. A., Grandhi, R. V., and Venkayya, V. B., "Optimum Design of Structures with Multiple Constraints," *AIAA Journal*, Vol. 26, No. 1, 1988, pp. 78–85.

[22]Canfield, R. A., Grandhi, R. V., and Venkayya, V. B., "Structural Optimization with Stiffness and Frequency Constraints," *Mechanics of Structures & Machines*, Vol. 17, No. 1, 1989, pp. 95–110.

[23]Canfield, R. A., "High-Quality Approximation of Eigenvalues in Structural Optimization," *AIAA Journal*, Vol. 28, No. 6, 1990, pp. 1116–1122.

[24]Canfield, R. A., "Design of Frames Against Buckling Using a Rayleigh Quotient Approximation," *AIAA Journal*, Vol. 31, No. 6, 1993, pp. 1143–1149.

[25]Cardou, A., "Piecewise Uniform Optimum Design for Axial Vibration Requirement," *AIAA Journal*, Vol. 11, No. 12, 1973, pp. 1760, 1761.

[26]Cardou, A., and Warner, W. H., "Minimum Mass Design of Sandwich Structures with Frequency and Section Constraints," *Journal of Optimization Theory and Applications*, Vol. 14, No. 6, 1974, pp. 633–647.

[27]Cassis, J. H., and Schmit, L. A., Jr., "On Implementation of the Extended Interior Penalty Function," *International Journal for Numerical Methods in Engineering*, Vol. 10, No. 1, 1976, pp. 3–23.

[28]Chattopadhyay, A., Hanagud, S. V., and Smith, C. V., Jr., "Minimum Weight Design of a Structure with Dynamic Constraints and a Coupling of Bending and Torsion," *Proceedings of the AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics, and Materials Conference* (San Antonio, TX), Pt. 2, AIAA, New York, 1986, pp. 386–394.

[29]Chattopadhyay, A., and Walsh, J. L., "Minimum Weight Design of Rotor Blades with Multiple Frequency and Stress Constraints," *AIAA Journal*, Vol. 28, No. 3, 1990, pp. 565–567.

[30]Chen, T.-Y., "Optimum Design of Structures with Both Natural Frequency and Frequency Response Constraints," *International Journal for Numerical Methods in Engineering*, Vol. 33, No. 9, 1992, pp. 1927–1940.

[31]Cheng, K., and Gu, Y., "Sequential Quadratic Programming and Dynamic Optimal Design of Rotating Blades," *Journal of Structural Mechanics*, Vol. 11, No. 4, 1983–84, pp. 451–464.

[32]Cheu, T.-C., Wang, B. P., and Chen, T.-Y., "Design Optimization of Gas Turbine Blades with Geometry and Natural Frequency Constraints," *Computers and Structures*, Vol. 32, No. 1, 1989, pp. 113–117.

[33]Choi, K. K., Haug, E. J., and Seong, H., "An Iterative Method for Finite Dimensional Structural Optimization Problems with Repeated Eigenvalues," *International Journal for Numerical Methods in Engineering*, Vol. 19, No. 1, 1983, pp. 93–112.

[34]Dailey, R. L., "Eigenvector Derivatives with Repeated Eigenvalues," *AIAA Journal*, Vol. 27, No. 4, 1989, pp. 486–491.

[35]Davis, M. W., and Weller, W. H., "Helicopter Rotor Dynamics Optimization with Experimental Verification," *Journal of Aircraft*, Vol. 28, No. 1, 1991, pp. 38–48.

[36]DeSilva, B. M. E., "Minimum Weight Design of Disks Using a Frequency Constraint," *ASME Journal of Engineering for Industry*, Vol. 91, Nov. 1969, pp. 1091–1099.

[37]DeSilva, B. M. E., and Grant, G. N. C., "Optimal Frequency-weight Computations for a Disk," *International Journal for Numerical Methods in Engineering*, Vol. 9, No. 3, 1971, pp. 387–393.

[38]Eldred, M., Venkayya, V., and Anderson, W., "Mode Tracking Issues in Optimization," AIAA Paper 93-1416, April 1993.

[39]Elwany, M. H. S., and Barr, A. D. S., "Some Optimization Problems in Torsional Vibration," *Journal of Sound and Vibration*, Vol. 57, No. 1, 1978, pp. 1–33.

[40]Elwany, M. H. S., and Barr, A. D. S., "Minimum Weight Design of Beams in Torsional Vibration with Several Frequency Constraints," *Journal of Sound and Vibration*, Vol. 62, No. 3, 1979, pp. 411–425.

[41]Elwany, M. H. S., and Barr, A. D. S., "Optimal Design of Beams Under Flexural Vibration," *Journal of Sound and Vibration*, Vol. 88, No. 2, 1983, pp. 175–195.

[42]Felix, J., and Vanderplaats, G. N., "Configuration Optimization of Trusses Subject to Strength, Displacement, and Frequency Constraints," *Proceedings of the 2nd International Computer Engineering Conference*

(San Diego, CA), Vol. 3, American Society of Mechanical Engineers, New York, 1982, pp. 109–118.

[43]Fleury, C., and Sander, G., "Generalized Optimality Criteria for Frequency Constraints, Buckling Constraints and Bending Elements," Air Force Office of Scientific Research, AFOSR-TR-80-0107, 1980.

[44]Fleury, C., and Braibant, V., "Structural Optimization—A New Dual Method Using Mixed Variables," *International Journal for Numerical Methods in Engineering,* Vol. 23, No. 3, 1986, pp. 409–428.

[45]Foley, M. H., "A Minimum Mass Square Plate with Fixed Fundamental Frequency of Free Vibration," *AIAA Journal*, Vol. 16, No. 9, 1978, pp. 1001–1004.

[46]Fox, R. L., and Kapoor, M. P., "Rates of Change of Eigenvalues and Eigenvectors," *AIAA Journal*, Vol. 6, No. 12, 1968, pp. 2426–2429.

[47]Fox, R. L., and Kapoor, M. P., "Structural Optimization in the Dynamics Response Regime: A Computational Approach," *AIAA Journal*, Vol. 8, No. 10, 1970, pp. 1798–1804.

[48]Friedmann, P. P., and Santhakumaran, P., "Optimum Design of Rotor Blades for Vibration Reduction in Forward Flight," *Journal of the American Helicopter Society*, Vol. 29, No. 4, 1984, pp. 70–80.

[49]Grandhi, R. V., and Venkayya, V. B., "Structural Optimization with Frequency Constraints," *AIAA Journal*, Vol. 26, No. 7, 1988, pp. 858–866.

[50]Grandhi, R. V., and Moradmand, J. K., "Optimum Synthesis of Thin-Walled Vibrating Beams with Coupled Bending and Torsion," *ASME Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 111, Dec. 1989, pp. 555–560.

[51]Grandhi, R. V., and Venkayya, V. B., "Optimum Design of Wing Structures with Multiple Frequency Constraints," *Finite Elements in Analysis and Design*, Vol. 4, 1989, pp. 303–313.

[52]Grandhi, R. V., "Optimum Design of Space Structures with Active and Passive Damping," *Engineering with Computers*, Vol. 6, No. 3, 1990, pp. 177–183.

[53]Grandhi, R. V., Bharatram, G., and Venkayya, V. B., "Optimum Design of Plate Structures with Multiple Frequency Constraints," *Journal of Structural Optimization*, Vol. 5, Nos. 1–2, 1992, pp. 100–107.

[54]Gura, N. M., and Seiranian, A. P., "Optimum Circular Plate with Constraints on the Rigidity and Frequency of Natural Oscillations," *Mechanics of Solids*, Vol. 12, No. 1, 1977, pp. 129–136.

[55]Haftka, R. T., and Grandhi, R. V., "Structural Shape Optimization—A Survey," *Computer Methods in Applied Mechanics and Engineering,* Vol. 57, No. 1, 1986, pp. 91–106.

[56]Haftka, R. T., Martinovic, Z. N., and Hallauer, W. L., Jr., "Enhanced Vibration Controllability by Minor Structural Modifications," *AIAA Journal*, Vol. 23, No. 8, 1991, pp. 1260–1266.

[57]Haftka, R. T., and Adelman, H. M., "Sensitivity of Discrete Systems," *Optimization of Large Structural Systems*, edited by G. I. N. Rozvany, Vol. 1, NATO ASI Series, Kluwer, The Netherlands, 1993, pp. 289–311.

[58]Hajela, P., and Shih, C. -J., "Multiobjective Optimum Design in Mixed Integer and Discrete Design Variable Problems," *AIAA Journal*, Vol. 28, No. 4, 1990, pp. 670–675.

[59]Hajela, P., Bloebaum, C. L., and Sobieski, J. S., "Application of Global Sensitivity Equations in Multidisciplinary Aircraft Synthesis," *Journal of Aircraft*, Vol. 27, No. 12, 1990, pp. 1002–1010.

[60]Hanagud, S., Smith, C. V., and Chattopadhyay, A., "Optimal Design of a Vibrating Beam with Coupled Bending and Torsion," *AIAA Journal*, Vol. 25, No. 9, 1987, pp. 1231–1240.

[61]Haug, E. J., Pan, K. C., and Streeter, T. D., "A Computational Method for Optimal Structural Design I: Piecewise Uniform Structures," *International Journal for Numerical Methods in Engineering*, Vol. 5, No. 2, 1972, pp. 171–184.

[62]Haug, E. J., Pan, K. C., and Streeter, T. D., "A Computational Method for Optimal Structural Design II: Continuous Problems," *International Journal for Numerical Methods in Engineering,* Vol. 9, May–June 1975, pp. 649–667.

[63]Haug, E. J., Arora, J. S., and Matsui, K., "A Steepest Descent Method for Optimization of Mechanical Systems," *Journal of Optimization Theory and Applications,* Vol. 19, No. 3, 1976, pp. 401–424.

[64]Haug, E. J., and Choi, K. K., "Systematic Occurrence of Repeated Eigenvalues in Structural Optimization," *Journal of Optimization Theory and Applications,* Vol. 38, No. 2, 1982, pp. 251–274.

[65]Hou, J. W., and Chuang, C. H., "Design Sensitivity Analysis and Optimization of Vibrating Beams with Variable Support Locations," *16th Design Automation Conference*, DE-Vol. 23-2, American Society of Mechanical Engineers, New York, 1990, pp. 281–290; also *ASME Journal of Mechanical Design* (to be published).

[66]Juang, J., Lim, K. B., and Junkins, J. L., "Robust Eigensystem Assignment for Flexible Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 3, 1989, pp. 381–387.

[67]Kamat, M. P., and Simitses, G. J., "Optimal Beam Frequencies by the Finite Element Displacement Method," *International Journal of Solids and Structures*, Vol. 9, March 1973, pp. 415–429.

[68]Kamat, M. P., "Effect of Shear Deformations and Rotary Inertia on Optimum Beam Frequencies," *International Journal for Numerical Methods in Engineering*, Vol. 9, No. 1, 1975, pp. 51–62.

[69]Kamat, M. P., Venkayya, V. B., and Khot, N. S., "Optimization with Frequency Constraints—Limitations," *Journal of Sound and Vibration*, Vol. 91, No. 1, 1983, pp. 147–154.

[70]Karihaloo, B. L., and Niordson, F. I., "Optimum Design of Vibrating Cantilevers," *Journal of Optimization Theory and Applications*, Vol. 11, No. 6, 1973, pp. 638–654.

[71]Karihaloo, B. L., and Parbery, R. D., "Minimum-Weight Multi-Constraint Vibrating Cantilevers," *International Journal of Solids and Structures*, Vol. 18, No. 5, 1982, pp. 419–430.

[72]Katarya, R., and Murthy, P. N., "Optimization of Multi-cell Wings for Strength and Natural Frequency Requirements," *Computers and Structures*, Vol. 5, 1975, pp. 225–232.

[73]Kegl, M. S., Butinar, B. J., and Oblak, M. M., "Optimization of Mechanical Systems: On Strategy of Non-Linear First-Order Approximation," *International Journal for Numerical Methods in Engineering,* Vol. 33, No. 2, 1992, pp. 223–234.

[74]Khan, M. R., and Willmert, K. D., "An Efficient Optimality Criterion Method for Natural Frequency Constrained Structures," *Computers and Structures,* Vol. 14, Nos. 5-6, 1981, pp. 501–507.

[75]Khot, N. S., "Optimal Design of a Structure for System Stability for a Specified Eigenvalue Distribution," *New Directions in Optimum Structural Design*, edited by E. Atrek, R. H. Gallagher, K. M. Ragsdell, and O. C. Zienkiewicz, Wiley, New York, 1984, pp. 75–87.

[76]Khot, N. S., "Optimization of Structures with Multiple Frequency Constraints," *Computers and Structures,* Vol. 20, No. 5, 1985, pp. 869–876.

[77]Khot, N. S., and Veley, D. E., "Robustness Characteristics of Optimum Structural/Control Design," *Journal of Guidance, Control, and Dynamics,* Vol. 15, No. 1, 1992, pp. 81–87.

[78]Kibsgaard, S., "Single and Multi-Purpose Optimization of Vibrating Timoshenko Shafts," *Journal of Structural Optimization,* Vol. 1, No. 2, 1989, pp. 91–100.

[79]Kiusalaas, J., and Shaw, R. C., "An Algorithm for Optimal Structural Design with Frequency Constraints," *International Journal for Numerical Methods in Engineering,* Vol. 13, 1978, pp. 283–295.

[80]Kodiyalam, S., Vanderplaats, G. N., and Miura, H., "Structural Shape Optimization with MSC/NASTRAN," *Computers and Structures,* Vol. 40, No. 4, 1991, pp. 821–829.

[81]Kryo'ko, V. A., and Pavlov, S. P., "Problem of Optimal Control of the Natural Frequency of Inhomogeneous Shells," *Soviet Applied Mechanics*, Vol. 18, No. 4, 1982, pp. 319–325.

[82]Leal, R. P., and Mota Soares, C. A., "Mixed Elements in the Optimal Design of Plates," *Journal of Structural Optimization*, Vol. 1, No. 2, 1989, pp. 127–136.

[83]Levy, R., and Chai, K., "Implementation of Natural Frequency Analysis and Optimality Criterion Design," *Computers and Structures,* Vol. 10, 1979, pp. 277–282.

[84]Lim, J. W., and Chopra, I., "Aeroelastic Optimization of a Helicopter Rotor Using an Efficient Sensitivity Analysis," *Journal of Aircraft*, Vol. 28, No. 1, 1991, pp. 29–37.

[85]Lin, C.-C., and Yu, A.-J., "Optimum Weight Design of Composite Laminated Plates," *Computers and Structures*, Vol. 38, Nos. 5/6, 1991, pp. 581–587.

[86]Lin, J. H., Che, W. Y., and Yu, Y. S., "Structural Optimization on Geometrical Configuration and Element Sizing with Statical and Dynamical Constraints," *Computers and Structures*, Vol. 15, No. 5, 1982, pp. 507–515.

[87]Livine, E., Schmit, L. A., and Friedmann, P. P., "Towards Integrated Multidisciplinary Synthesis of Actively Controlled Fiber Composite Wings," *Journal of Aircraft*, Vol. 27, No. 12, 1990, pp. 979–992.

[88]Lust, R. V., and Schmit, L. A., "Control-Augmented Structural Synthesis," *AIAA Journal*, Vol. 26, No. 1, 1988, pp. 86–94.

[89]Maghami, P. G., and Juang, J.-N., "Efficient Eigenvalue Assignment for Large Space Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, 1990, pp. 1033–1039.

[90]Masur, E. F., "Optimal Placement of Available Sections in Structural Eigenvalue Problems," *Journal of Optimization Theory and Applications*, Vol. 15, No. 1, 1975, pp. 69–84.

[91]Masur, E. F., "Optimal Structural Design Under Multiple Eigenvalue Constraints," *International Journal of Solids and Structures*, Vol. 20, No. 3, 1984, pp. 211–231.

[92]Masur, E. F., "Some Additional Comments on Optimal Structural Design Under Multiple Eigenvalue Constraints," *International Journal of Solids and Structures*, Vol. 21, No. 2, 1985, pp. 117–120.

[93]Mateus, H. C., Mota Soares, C. M., and Mota Soares, C. A., "Sensitivity Analysis and Optimal Design of Thin Laminated Composite Structures," *Computers and Structures*, Vol. 41, No. 3, 1991, pp. 501–508.

[94]McCart, B. R., Haug, E. J., and Streeter, T. D., "Optimal Design of Structures with Constraints on Natural Frequency," *AIAA Journal*, Vol. 8,

No. 6, 1970, pp. 1012–1019.
[95]McConnell, R. D., "Least Weight Structures for Threshold Frequencies in a Seismic Environment," *Computers and Structures*, Vol. 7, Nov. 1977, pp. 157–160.
[96]McGee, O. G., and Phan, K. F., "A Robust Optimality Criteria Procedure for Cross-Sectional Optimization of Frame Structures with Multiple Frequency Limits," *Computers and Structures*, Vol. 38, Nos. 5/6, 1991, pp. 485–500.
[97]McGee, O. G., and Phan, K. F., "Adaptable Optimality Criterion Techniques for Large Scale Space Frames with Multiple Frequency Constraints," *Computers and Structures*, Vol. 42, No. 2, 1992, pp. 197–210.
[98]Melnikov, Y. A., and Titarenko, S. A., "A New Approach to 2-D Eigenvalue Shape Design," *International Journal for Numerical Methods in Engineering*, Vol. 36, No. 12, 1993, pp. 2017–2030.
[99]Mesquita, L. C., and Kamat, M. P., "Optimal Design of Stiffened Laminated Composite Plates with Frequency Constraints," *Engineering Optimization*, Vol. 11, 1987, pp. 77, 78.
[100]Miele, A., Mangiavacchi, Mohanty, B. P., and Wu, A. K., "Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies," *International Journal for Numerical Methods in Engineering*, Vol. 13, April 1978, pp. 265–282.
[101]Mills-Curran, W. C., "Calculation of Eigenvector Derivatives for Structures with Repeated Eigenvalues," *AIAA Journal*, Vol. 26, No. 7, 1988, pp. 867–871.
[102]Miura, H., and Schmit, L. A., Jr., "Second Order Approximation of Natural Frequency Constraints in Structural Synthesis," *International Journal for Numerical Methods in Engineering*, Vol. 13, April 1978, pp. 337–351.
[103]Mlejnek, H. P., Jehle, U., and Schirrmacher, R., "Second Order Approximations in Structural Genesis and Shape Finding," *International Journal for Numerical Methods in Engineering*, Vol. 34, No. 3, 1992, pp. 853–872.
[104]Murthy, T. S., "Optimization of Helicopter Airframe Structures for Vibration Reduction—Considerations, Formulations, and Applications," *Journal of Aircraft*, Vol. 28, No. 1, 1991, pp. 66–73.
[105]Myslinski, A., "Bimodal Optimal Design of Vibrating Plates Using Theory and Methods of Nondifferentiable Optimization," *Journal of Optimization Theory and Applications*, Vol. 46, No. 2, 1985, pp. 187–203.
[106]Nakamura, T., and Ohsaki, M., "A Natural Generation of Optimum Topology of Plane Trusses for Specified Fundamental Frequency," *Computer Methods in Applied Mechanics and Engineering*, Vol. 94, No. 1, 1992, pp. 113–129.
[107]Neill, D. J., Johnson, E. H., and Canfield, R., "ASTROS—A Multidisciplinary Automated Structural Design Tool," *Journal of Aircraft*, Vol. 27, No. 12, 1990, pp. 1021–1027.
[108]Nelson, R. B., "Simplified Calculation of Eigenvector Derivatives," *AIAA Journal*, Vol. 14, No. 9, 1976, pp. 1201–1205.
[109]Ngyuen, D. T., and Arora, J. S., "Fail-Safe Optimal Design of Complex Structures with Substructures," *ASME Journal of Mechanical Design*, Vol. 104, Oct. 1982, pp. 861–868.
[110]Niordson, F. I., "On the Optimal Design of a Vibrating Beam," *Quarterly of Applied Mathematics*, Vol. 23, No. 1, 1965, pp. 47–53.
[111]Ojalvo, I. U., "Efficient Computation of Mode-Shape Derivatives for Large Dynamic Systems," *AIAA Journal*, Vol. 25, No. 10, 1987, pp. 1386–1390.
[112]Olhoff, N., "Optimal Design of Vibrating Rectangular Plates," *International Journal of Solids and Structures*, Vol. 10, No. 1, 1974, pp. 93–109.
[113]Olhoff, N., "Optimization of Vibrating Beams with Respect to Higher Order Natural Frequencies," *Journal of Structural Mechanics*, Vol. 4, No. 1, 1976, pp. 87–122.
[114]Olhoff, N., "A Survey of the Optimal Design of Vibrating Structural Elements. Part 1: Theory," *Shock and Vibration Digest*, Vol. 8, No. 2, 1976, pp. 3–10.
[115]Olhoff, N., "A Survey of the Optimal Design of Vibrating Structural Elements. Part 2: Applications," *Shock and Vibration Digest*, Vol. 8, No. 9, 1976, pp. 3–10.
[116]Olhoff, N., "Maximizing Higher Order Eigen Frequencies of Beams with Constraints on the Design Geometry," *Journal of Structural Mechanics*, Vol. 5, No. 2, 1977, pp. 107–134.
[117]Olhoff, N., "Optimization of Transversely Vibrating Beams and Rotating Shafts," *Optimization of Distributed Parameter Structures*, edited by by E. J. Haug and J. Cea, Sijthoff and Noordoff, Alphen aan den Rihn, The Netherlands, 1980.
[118]Olhoff, N., "Optimal Design with Respect to Structural Eigenvalues," *Theoretical and Applied Mechanics: Proceedings of the Fifteenth International Congress* (Toronto, Canada), Aug. 1980, pp. 133–149.
[119]Olhoff, N., and Plaut, R. H., "Bimodal Optimization of Vibrating Shallow Arches," *International Journal of Solids and Structures*, Vol. 19, No. 6, 1983, pp. 553–570.
[120]Olhoff, N., and Parbery, R., "Designing Vibrating Beams and Rotating Shafts for Maximum Difference Between Adjacent Natural Frequencies," *International Journal of Solids and Structures*, Vol. 20, No. 1, 1984, pp. 63–75.
[121]Pan, T. S., Rao, S. S., and Venkayya, V. B., "Rates of Change of Closed-Loop Eigenvalues and Eigenvectors of Actively Controlled Structures," *International Journal for Numerical Methods in Engineering*, Vol. 30, March 1990, pp. 1013–1028.
[122]Pappas, M., and Amba Rao, C. L., "A Direct Search Algorithm for Automated Optimum Structural Design," *AIAA Journal*, Vol. 9, No. 3, 1971, pp. 387–393.
[123]Pappas, M., "Optimal Frequency Separation of Cylindrical Shells," *AIAA Journal*, Vol. 16, No. 9, 1978, pp. 999–1001.
[124]Parbery, R. D., "Minimum Weight Bars for Given Lower Bounds on Euler Buckling Load and Frequency of Longitudinal Vibration," *Computers and Structures*, Vol. 23, No. 8, 1987, pp. 1163–1178.
[125]Patnaik, S. N., and Maiti, M., "Optimum Design of Stiffened Structures with Constraint on the Frequency in the Presence of Initial Stresses," *Computer Methods in Applied Mechanics and Engineering*, Vol. 7, 1976, pp. 303–322.
[126]Pedersen, P., "Design with Several Eigenvalue Constraints by Finite Elements and Linear Programming," *Computers and Structures*, Vol. 10, No. 3, 1982–83, pp. 243–271.
[127]Peters, D. A., Ko, T., Korn, A., and Rossow, M. P., "Design of Helicopter Rotor Blades for Desired Placement of Natural Frequencies," *Proceedings of the 39th Annual Forum of the American Helicopter Society* (St. Louis, MO), American Helicopter Society, 1983.
[128]Phan, K. F., and McGee, O. G., "An Adaptive Procedure for Stabilizing Convergence Quality in Frequency-Constrained Design Optimization of Bar Structures Supporting Nonstructural Mass," *International Journal for Numerical Methods in Engineering*, Vol. 32, No. 1, 1991, pp. 1–27.
[129]Pierson, B. L., "A Survey of Optimal Structural Design Under Dynamic Constraints," *International Journal for Numerical Methods in Engineering*, Vol. 4, 1972, pp. 491–499.
[130]Pierson, B. L., "An Optimal Control Approach to Minimum Weight Vibrating Beam Design," *Computers and Structures*, Vol. 5, No. 2, 1977, pp. 147–178.
[131]Pierson, B. L., and Pouliot, M. R., "Minimum-Weight Design of a Rotating Cantilever Beam with Specified Flapping Frequency," *Optimization of Distributed Parameter Structures*, edited by E. J. Haug and J. Cea, Sijthoff & Noordhoff, Alphen aan den Rijn, The Netherlands, 1981.
[132]Plaut, R. H., and Olhoff, N., "Optimal Forms of Shallow Arches with Respect to Vibration and Stability," *Journal of Structural Mechanics*, Vol. 11, 1983, pp. 81–100.
[133]Plaut, R. H., Johnson, L. W., and Parbery, R., "Optimal Forms of Shallow Shells with Circular Boundary, Part 1: Maximum Fundamental Frequency," *ASME Journal of Applied Mechanics*, Vol. 51, Sept. 1984, pp. 526–530.
[134]Prager, W., and Taylor, J. E., "Problems of Optimal Structural Design," *Journal of Applied Mechanics*, Vol. 35, March 1968, pp. 102–106.
[135]Pritchard, J. I., Adelman, H. M., and Haftka, R. T., "Sensitivity Analysis and Optimization of Nodal Point Placement for Vibration Reduction," *Journal of Sound and Vibration*, Vol. 119, No. 2, 1987, pp. 277–289.
[136]Pritchard, J. I., and Adelman, H. M., "Differential Equation Based Method for Accurate Modal Approximations," *AIAA Journal*, Vol. 29, No. 3, 1991, pp. 484–486.
[137]Purtov, V. A., and Pshenichnov, G. I., "Optimal Design of a Spherical Net-Like Shell with a Fixed First Eigenfrequency of Axisymmetric Oscillations," *Journal of Applied Mathematics and Mechanics*, Vol. 45, 1982, pp. 672–676.
[138]Rangacharyulu, M. A. V., and Done, G. T. S., "A Survey of Structural Optimization Under Dynamic Constraints," *Shock and Vibration Digest*, Vol. 11, No. 12, 1979, pp. 15–25.
[139]Rao, S. S., "Optimization of Complex Structures to Satisfy Static, Dynamic, and Aeroelastic Requirements," *International Journal for Numerical Methods in Engineering*, Vol. 8, No. 2, 1973, pp. 249–269.
[140]Rao, S. S., and Singh, K. K., "Optimum Design of Laminates with Natural Frequency Constraints," *Journal of Sound and Vibration*, Vol. 67, No. 1, 1979, pp. 101–112.
[141]Rao, S. S., and Reddy, E. S., "Optimum Design of Stiffened Cylindrical Shells with Natural Frequency Constraints," *Computers and Structures*, Vol. 12, Aug. 1980, pp. 211–219.
[142]Rao, S. S., and Reddy, E. S., "Optimum Design of Stiffened Conical Shells with Natural Frequency Constraints," *Computers and Structures*, Vol. 14, Nos. 1–2, 1981, pp. 103–110.
[143]Rao, V. R., Iyengar, N. G. R., and Rao, S. S., "Optimization of Wing Structures to Satisfy Strength and Frequency Requirements," *Computers and Structures*, Vol. 10, No. 4, 1979, pp. 669–674.
[144]Reddy, C. P., and Rao, S. S., "Automated Optimum Design of Machine Tool Structures for Static Rigidity, Natural Frequencies, and Regenerative Chatter Stability," *ASME Journal of Engineering for Industry*, Vol.

100, No. 2, 1978, pp. 137–146.

[145]Rew, D. W., Junkins, J. L., and Juang, J. N., "Robust Eigenstructure Assignment by a Projection Method: Applications using Multiple Optimization Criteria," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 3, 1989, pp. 396–403.

[146]Rizzi, P., "Optimization of Multi-Constrained Structures Based on Optimality Criteria," *Proceedings of the 17th Structures, Structural Dynamics, and Materials Conference Proceedings* (King of Prussia, PA), AIAA, New York, 1976, pp. 448–462.

[147]Rubin, C. P., "Minimum Weight Design of Complex Structures Subject to a Frequency Constraint," *AIAA Journal*, Vol. 8, No. 5, 1970, pp. 923–927.

[148]Sadek, E. A., "Dynamic Optimization of Framed Structures," *Computers and Structures*, Vol. 21, No. 6, 1985, pp. 1313–1323.

[149]Sadek, E. A., "Dynamic Optimization of Framed Structures with Variable Layout," *International Journal for Numerical Methods in Engineering*, Vol. 23, No. 7, 1986, pp. 1273–1294.

[150]Sadek, E. A., "An Optimality Criterion Method for Dynamic Optimization of Structures," *International Journal for Numerical Methods in Engineering*, Vol. 28, No. 3, 1989, pp. 579–592.

[151]Sepulveda, A. E., and Schmit, L. A., Jr., "Optimal Placement of Actuators and Sensors in Control-Augmented Structural Optimization," *International Journal for Numerical Methods in Engineering*, Vol. 32, No. 6, 1991, pp. 1165–1187.

[152]Shamie, J., and Schmit, L. A., "Frame Optimization Including Frequency Constraints," *Journal of the Structural Division*, ST1, 1975, pp. 283–291.

[153]Sheu, C. Y., "Elastic Minimum Weight Design for Specified Fundamental Frequency," *International Journal of Solids and Structures*, Vol. 4, No. 10, 1968, pp. 953–958.

[154]Shin, Y. S., Haftka, R. T., and Plaut, R. H., "Simultaneous Analysis and Design for Eigenvalue Maximization," *AIAA Journal*, Vol. 26, No. 6, 1988, pp. 738–744.

[155]Sippel, D. L., and Warner, W. H., "Minimum-Mass Design of Multielement Structures under a Frequency Constraint," *AIAA Journal*, Vol. 11, No. 4, 1973, pp. 483–489.

[156]Smith, M. J., and Hutton, S. G., "Frequency Modification Using Newton's Method and Inverse Iteration Eigenvector Updating," *AIAA Journal*, Vol. 30, No. 7, 1992, pp. 1886–1891.

[157]Sobieszczanski-Sobieski, J., "Structural Optimization: Challenges and Opportunities," *International Journal of Vehicle Design*, Vol. 7, 1986, pp. 242–263.

[158]Somayajula, G., and Bernard, J., "Design Optimization of Structures Subject to Static and Dynamic Constraints," *Finite Elements in Analysis and Design*, Vol. 5, 1989, pp. 281–289.

[159]Spillers, W. R., Singh, S., and Levy, R., "Optimization with Frequency Constraint," *Journal of the Structural Division, ASCE*, Vol. 107, No. ST12, Dec. 1981, pp. 2337–2347.

[160]Starnes, J. H., Jr., and Haftka, R. T., "Preliminary Design of Composite Wings for Buckling, Stress, and Displacement Constraints," *Journal of Aircraft*, Vol. 16, No. 8, 1979, pp. 564–570.

[161]Sunar, M., and Belegundu, A. D., "Trust Region Methods for Structural Optimization Using Exact Second Order Sensitivity," *International Journal for Numerical Methods in Engineering*, Vol. 32, No. 2, 1991, pp. 275–293.

[162]Sutter, T. R., Camarda, C. J., Walsh, J. L., and Adelman, H. M., "Comparison of Several Methods for Calculating Vibration Mode Shape Derivatives," *AIAA Journal*, Vol. 26, No. 12, 1988, pp. 1506–1511.

[163]Szymczak, C., "Optimal Design of Thin Walled I Beams for Extreme Natural Frequency of Torsional Vibrations," *Journal of Sound and Vibration*, Vol. 86, No. 2, 1983, No. 235–241.

[164]Szymczak, C., "Optimal Design of Thin Walled I Beams for a Given Natural Frequency of Torsional Vibrations," *Journal of Sound and Vibration*, Vol. 97, No. 1, 1984, pp. 137–144.

[165]Szyszkowski, W., "Multimodal Optimality Criterion for Maximum Fundamental Frequency of Free Vibrations," *Computers and Structures*, Vol. 41, No. 5, 1991, pp. 909–916.

[166]Taig, I. C., "Optimization of Aircraft Structures with Multiple Stiffness Requirements," Second Symposium on Structural Optimization, AGARD-CP-123, Milan, Italy, April 1973.

[167]Taylor, J. E., "Minimum Mass Bar for Axial Vibration at Specified Natural Frequency," *AIAA Journal*, Vol. 5, No. 10, 1967, pp. 1911–1913.

[168]Taylor, J. E., "Optimum Design of a Vibrating Bar with Specified Minimum Cross-section," *AIAA Journal*, Vol. 6, No. 7, 1968, pp. 1379–1381.

[169]Thomas, H. L., Sepulveda, A. E., and Schmit, L. A., Jr., "Improved Approximations for Control Augmented Structural Synthesis," *AIAA Journal*, Vol. 30, No. 1, 1992, pp. 171–179.

[170]Thomas, H. L., Shyy, Y.-K., Vanderplaats, G. N., and Miura, H., "GENESIS—A Modern Shape and Sizing Structural Design Tool," AIAA Paper 92-2559, April 1992.

[171]Ting, T., Chen, T., and Twomey, W., "An Automated Mode-Tracking Strategy," AIAA Paper 93-1414, 1993.

[172]Tseng, C. H., and Kao, K. Y., "Performance of a Hybrid Sensitivity Analysis in Structural Design Problems," *Computers and Structures*, Vol. 33, No. 5, 1989, pp. 1125–1131.

[173]Tseng, C. H., and Lu, T. W., "Minimax Multiobjective Optimization in Structural Design," *International Journal for Numerical Methods in Engineering*, Vol. 30, No. 6, 1990, pp. 1213–1228.

[174]Turner, M. J., "Design of Minimum Mass Structures with Specified Natural Frequencies," *AIAA Journal*, Vol. 5, No. 3, 1967, pp. 406–412.

[175]Vanderplaats, G. N., and Salajegheh, E., "An Efficient Approximation Technique for Frequency Constraints in Frame Optimization," *International Journal for Numerical Methods in Engineering*, Vol. 26, No. 9, 1988, pp. 1057–1069.

[176]Vavrick, D. J., and Warner, W. H., "Minimum Mass Design with Torsional Frequency and Thickness Constraints," *Journal of Structural Mechanics*, Vol. 6, No. 2, 1978, pp. 211–231.

[177]Venkayya, V. B., Khot, N. S., and Berke, L., "Application of Optimality Criteria Approaches to Automated Design of Large Practical Structures," Second Symposium on Structural Optimization, AGARD-CP-123, 1973, pp. 3-1–3-19.

[178]Venkayya, V. B., and Tischler, V. A., "Optimization of Structures with Frequency Constraints," *Computer Methods for Nonlinear Solids and Structural Mechanics, ASME*, AMD-54, June 1983, pp. 239–251.

[179]Venkayya, V. B., and Tischler, V. A., "Frequency Control and its Effect on the Dynamic Response of Flexible Structures," *AIAA Journal*, Vol. 23, No. 11, 1983, pp. 1768–1774.

[180]Venkayya, V. B., "Mathematical Optimization in Multi-Disciplinary Design," *Mathematical and Computer Modelling*, Vol. 14, 1990, pp. 29–36.

[181]Wang, B. P., Chang, Y. K., Lawrence, K. L., and Chen, T. Y., "Optimum Design of Structures with Multiple Configurations with Frequency and Displacement Constraints," AIAA Paper 90-1136, 1990, pp. 378–384.

[182]Wang, B. P., "Closed Form Solution for Minimum Weight Design with a Frequency Constraint," *AIAA Journal*, Vol. 29, No. 1, 1991, pp. 152–154.

[183]Warner, W. H., and Vavrick, D. J., "Optimal Design in Axial Motion for Several Frequency Constraints," *Journal of Optimization Theory and Applications*, Vol. 15, No. 1, 1975, pp. 157–166.

[184]Washizu, K., and Hanaoka, M., "Application of the Finite Element Method to Minimum Mass Design of a Bar with Two Specified Natural Frequencies," *Computers and Structures*, Vol. 10, 1979, pp. 539–545.

[185]Weisshaar, T. A., "Optimization of Simple Structures with Higher Mode Frequency Constraints," *AIAA Journal*, Vol. 10, No. 5, 1971, pp. 691–693.

[186]Weisshaar, T. A., "Approximate Solutions to Idealised Structural Dynamic Optimization Problems," *Journal of Optimization Theory and Applications*, Vol. 16, Nos. 1–2, 1975, pp. 119–133.

[187]Woo, T. H., "Space Frame Optimization Subject to Frequency Constraints," *AIAA Journal*, Vol. 25, No. 10, 1987, pp. 1396–1404.

[188]Young, J. W., and Christiansen, H. N., "Synthesis of a Space Truss Based on Dynamics Criteria," *Journal of the Structural Division, ASCE*, Vol. 92, No. ST6, Dec. 1966.

[189]Zacharopoulos, A., Willmert, K. D., and Khan, M. R., "An Optimality Criterion Method for Structures with Stress, Displacement, and Frequency Constraints," *Computers and Structures*, Vol. 19, No. 4, 1984, pp. 621–629.

[190]Zarghammee, M. S., "Optimum Frequency of Structures," *AIAA Journal*, Vol. 6, No. 4, 1967, pp. 749, 750.